

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
FIRST SEMESTER B.TECH DEGREE EXAMINATION(2019 SCHEME), DECEMBER 2019

**Course Code: MAT101**

**Course Name: LINEAR ALGEBRA AND CALCULUS**

(2019-Scheme)

Max. Marks: 100

Duration: 3 Hours

**PART A**

*Answer all questions, each carries 3 marks.*

- 1 Determine the rank of the matrix  $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & 5 \end{bmatrix}$  (3)
- 2 If 2 is an eigen value of  $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ , without using its characteristic equation, find the other eigen values. (3)
- 3 If  $f(x, y) = xe^{-y} + 5y$  find the slope of  $f(x, y)$  in the x-direction at (4,0). (3)
- 4 Show that  $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$ , where  $z = e^x \sin y + e^y \cos x$  (3)
- 5 Find the mass of the square lamina with vertices (0,0) (1,0) (1,1) and (0,1) and density function  $x^2 y$  (3)
- 6 Evaluate  $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)} dx dy$  by changing to polar coordinates. (3)
- 7 Test the convergence of the series  $\sum_{k=1}^{\infty} \frac{k}{2k+1}$  (3)
- 8 Check the convergence of  $\sum_{k=1}^{\infty} \frac{1}{k^{k/2}}$  (3)
- 9 Find the Taylors series for  $f(x) = \cos x$  about  $x = \frac{\pi}{2}$  up to third degree terms. (3)
- 10 Find the Fourier half range sine series of  $f(x) = e^{-x}$  in  $0 < x < 1$  (3)

**PART B**

*Answer one full question from each module, each question carries 14 marks*

**Module-I**

- 11 a) Solve the system of equations by Gauss elimination method. (7)

$$x + 2y + 3z = 1$$

$$2x + 3y + 2z = 2$$

$$3x + 3y + 4z = 1$$

- b) Find the eigenvalues and eigenvectors of (7)

$$\begin{bmatrix} 4 & 2 & -2 \\ 2 & 5 & 0 \\ -2 & 0 & 3 \end{bmatrix}$$

- 12 a) Find the values of  $\lambda$  and  $\mu$  for which the system of equations (7)

$$2x + 3y + 5z = 9$$

$$7x + 3y - 2z = 8$$

$$2x + 3y + \lambda z = \mu$$

has (i) no solution (ii) a unique solution and (iii) infinite solution

- b) Find the matrix of transformation that diagonalize the matrix (7)

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}. \text{ Also write the diagonal matrix.}$$

**Module-II**

- 13 a) Let  $f$  be a differentiable function of three variables and suppose that (7)

$$w = f(x - y, y - z, z - x), \text{ show that } \frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$$

- b) Locate all relative extrema of  $f(x, y) = 4xy - y^4 - x^4$  (7)

- 14 a) Find the local linear approximation  $L$  to the function  $f(x, y) = \sqrt{x^2 + y^2}$  (7)  
at the point  $P(3,4)$ . Compare the error in approximating  $f$  by  $L$  at the point  $Q(3.04, 3.98)$  with the distance  $PQ$ .

- b) The radius and height of a right circular cone are measured with errors of at most 1% and 4%, respectively. Use differentials to approximate the maximum percentage error in the calculated volume. (7)

**Module-III**

- 15 a) Evaluate  $\iint_R y dx dy$  where  $R$  is the region bounded by the parabolas  $y^2 = 4x$  and  $x^2 = 4y$ . (7)
- b) Use double integral to find the area of the region enclosed between the parabola  $y = \frac{x^2}{2}$  and the line  $y = 2x$ . (7)
- 16 a) Evaluate  $\int_0^1 \int_{\frac{y}{2}}^1 e^{x^2} dx dy$  by reversing the order of integration (7)
- b) Use triple integrals to find the volume of the solid within the cylinder  $x^2 + y^2 = 9$  and between the planes  $z = 1$  and  $x + z = 5$ . (7)

#### Module-IV

- 17 a) Find the general term of the series  $1 + \frac{1 \cdot 2}{1 \cdot 3} + \frac{1 \cdot 2 \cdot 3}{1 \cdot 3 \cdot 5} + \frac{1 \cdot 2 \cdot 3 \cdot 4}{1 \cdot 3 \cdot 5 \cdot 7} + \dots$  and use the ratio test to show that the series converges. (7)
- b) Test whether the following series is absolutely convergent or conditionally convergent  $\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k(k+1)}}$  (7)
- 18 a) Test the convergence of  $\frac{x}{1 \cdot 2} + \frac{x^2}{2 \cdot 3} + \frac{x^3}{3 \cdot 4} + \dots + \frac{x^k}{k(k+1)} + \dots$  (7)
- b) Test the convergence of the series  $\sum_{k=1}^{\infty} \frac{(k+1)!}{4! k! 4^k}$  (7)

#### Module-V

- 19 a) Find the Fourier series of periodic function with period 2 which is given below  $f(x) = \begin{cases} -x & ; -1 \leq x \leq 0 \\ x & ; 0 \leq x \leq 1 \end{cases}$ . Hence prove that  $1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$  (7)
- b) Find the half range cosine series for  $f(x) = \begin{cases} kx & 0 \leq x \leq L/2 \\ k(L-x) & L/2 \leq x \leq L \end{cases}$  (7)

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a) Find the Fourier series of  $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2 & 0 < x < \pi \end{cases}$  (7)

b) Obtain the Fourier series expansion for  $f(x) = x^2$ ,  $-\pi < x < \pi$ . (7)

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