

Reg No.: \_\_\_\_\_

Name: \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY**  
**FIFTH SEMESTER B.TECH DEGREE EXAMINATION(R&S), DECEMBER 2019**

**Course Code: CS309**

**Course Name: GRAPH THEORY AND COMBINATORICS**

Max. Marks: 100

Duration: 3 Hours

**PART A**

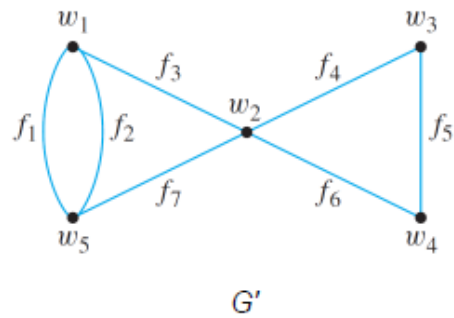
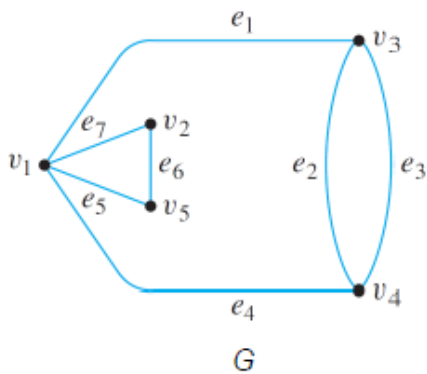
*Answer all questions, each carries 3 marks.*

- |   |   | Marks |
|---|---|-------|
| 1 | Define the terms a) Walk b) Path and c) Circuit with an example.  | (3)   |
| 2 | Prove that the no of vertices of odd degree in a graph is always even   | (3)   |
| 3 | Draw a graph that has a Hamiltonian path but does not have a Hamiltonian circuit.   | (3)   |
| 4 | Differentiate between Symmetric and Asymmetric digraphs with examples and draw a complete symmetric digraph of four vertices. | (3)   |

**PART B**

*Answer any two full questions, each carries 9 marks.*

- |   |   |     |
|---|---|-----|
| 5 | a) Prove that a simple graph with n vertices and k components can have at most $(n-k)(n-k+1)/2$ edges | (4) |
|   | b) Define Isomorphism of graphs. Check whether the two graphs are isomorphic or not                   | (5) |



- |   |   |     |
|---|---|-----|
| 6 | a) Define Euler graph. Check whether the graph is an euler graph or not. If yes, give the Euler line and justify your answer. | (5) |
|---|---|-----|



- |  |   |     |
|--|---|-----|
|  | b) Prove that a connected graph G is an Euler graph if and only if all vertices of G are of | (4) |
|--|---|-----|

even degree.

- 7 a) State travelling salesman problem and how TSP solution is related to Hamiltonian circuits. (5)
- b) State and Prove Dirac's Theorem for Hamiltonicity. (4)

### PART C

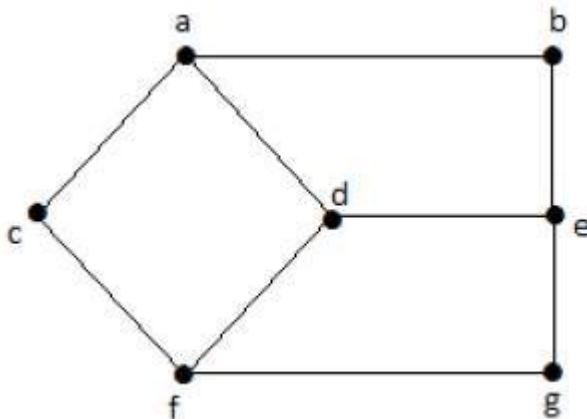
*Answer all questions, each carries 3 marks.*

- 8 Prove that the distance between the vertices of a connected graph is a metric (3)
- 9 List down any two properties of a tree and also prove the following theorem: A graph is a tree if and only if it is minimally connected. (3)
- 10 Define the terms vertex connectivity and edge connectivity with examples. (3)
- 11 Give the different representations of a planar graph. (3)

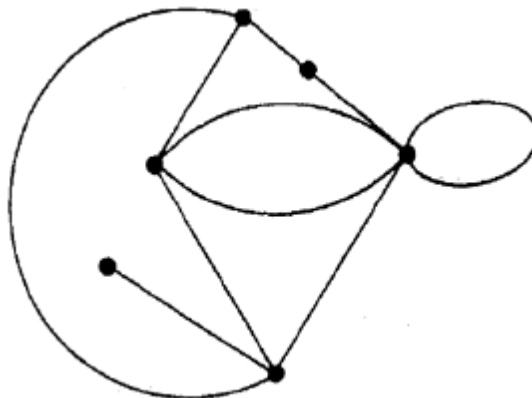
### PART D

*Answer any two full questions, each carries 9 marks.*

- 12 a) Find the eccentricity of all vertices in the graph G given below and also mark the center, radius and diameter of G (6)

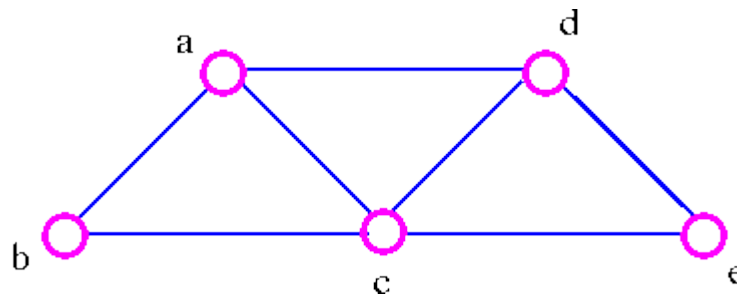


- b) State and prove Cayley's theorem (3)
- 13 a) Find the Geometrical dual ( $G^*$ ) of the graph G given below (5)



- b) List out the properties stating the relationship between the graph G and its dual  $G^*$  (4)

- 14 a) Consider the graph G and any one of its spanning tree T. Find all fundamental circuits and fundamental cut sets with respect to the spanning tree T. (6)



- b) Prove that “Every cut set in a connected graph G must contain atleast one branch of every spanning tree of G”. (3)

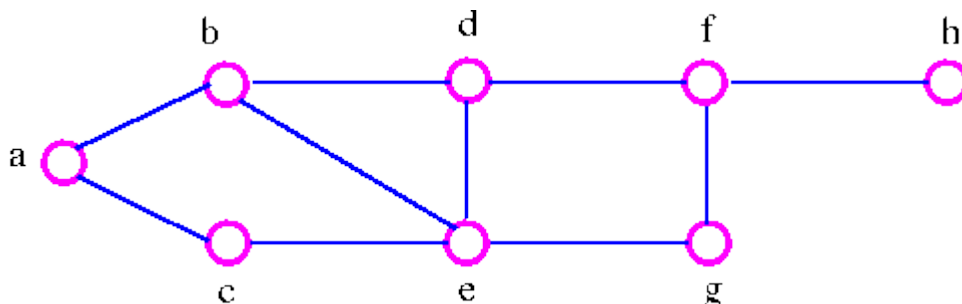
### PART E

*Answer any four full questions, each carries 10 marks.*

- 15 a) Define Adjacency Matrix  $X(G)$  of a graph. Determine the properties of adjacency matrix. (6)
- b) Draw the graph represented by the following adjacency matrix. (4)

$$X(G) = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}$$

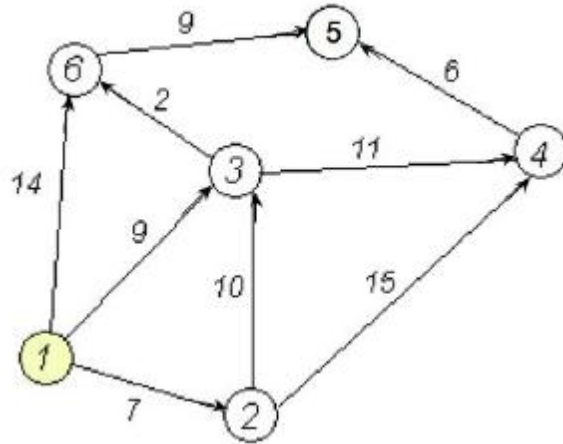
- 16 a) Obtain a cut set matrix for the following graph (6)



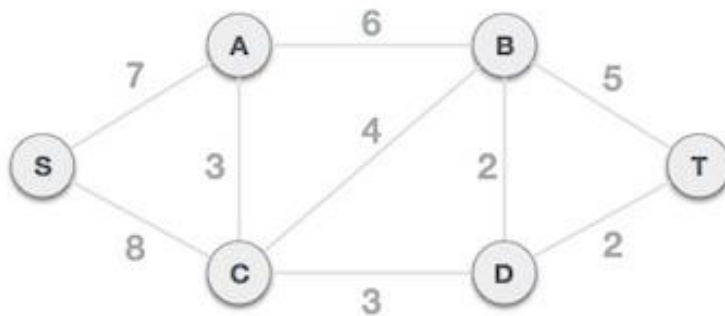
- b) Define path matrix. Determine the properties of a path matrix. (4)
- 17 a) Explain edge listing and successor listing methods used in computer representation of graphs (4)
- b) Draw the flow chart to determine connectedness and components of a graph (6)
- 18 Draw a flowchart indicating all the five conditions to find the spanning tree /spanning (10)

forest. Apply it to find the spanning tree /spanning forest for any graph of your choice.

- 19 Write Dijkstra's Shortest path algorithm and apply this algorithm to find the shortest path (10)  
path



- 20 Write Kruskal's algorithm to find the minimum spanning tree of a graph G. Apply it (10)  
to find the MST for the graph given below



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