Reg No.:_______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

THIRD SEMESTER B.TECH DEGREE EXAMINATION(R&S), DECEMBER 2019

Course Code: CS201

Course Name: DISCRETE COMPUTATIONAL STRUCTURES

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries3 marks. Marks

1 Let f(x) = x+2 g(x) = x-2 and h(x) = 3x for x ÎR, where R is the set of real numbers. Find foh, hog , fo(goh) (3)

2 Let R be a relation on set A .Prove that if R is reflexive then R^{-1} is also reflexive (3)

3 In how many ways can 12 balloons be distributed at a birthday party among 10 children if we ensure that every child gets atleast one balloon. (3)

4 Show that the set N of natural numbers is a monoid under the operation x* y= max(x,y). (3)

PART B

Answer any two full questions, each carries9 marks.

5 a) Prove that (i) A ⊕ B or (A- B) U (B – A) = (A UB) - (A ∩B) (5)

(ii) (A- C) U (B – C) = (A ∩B) – C Where A, B and C are any sets.

b) How many permutations can be made with the letters of the word MISSISSIPPI taken all together? How many of these will be vowels occupying the even places? (4)

6 a) Let X ={ 2,3,6,12,24,36} and the relation be such that x ≤y if x divides y . Give the relation and draw the Hasse diagram of (X, ≤). Also find the maximal and minimal elements. (4)

b) Solve the recurrence relation $a_r + a_{r-1} = 3r\, 2^r$ using characteristic root method (5)

7 a) State Pigeon hole principle. Show that among any 11 numbers there existat least 2 numbers with the same unit digit. (4)

b) Let A={1,2,3....11,12} and let R be the relation on A × A defined by (a,b) R (5) (c,d) iffa+d = b+c

(i) Prove that R is an equivalence relation
(ii) Find the equivalence class of (2,5)

PART C

Answer all questions, each carries3 marks.

8 Show that the order of a subgroup of a finite group divides the order of the group. (3)

9 Prove that the set consisting of the fourth roots of unity forms an abelian group with respect to multiplication composition. (3)

10 In a distributive lattice $a \vee b = a \vee c$ and $a \wedge b = a \wedge c$ imply that $b = c$. (3)

11 Show that the complement of every element in a boolean algebra is unique. (3)

PART D

Answer any two full questions, each carries9 marks.

12 a) The necessary and sufficient condition that a non-empty subset H of a Group G be a subgroup is $a \in H,\ \ b \in H \Rightarrow ab^{-1} \in H$ (5)

b) Let x, y be arbitrary elements in a boolean algebra (B, +, . ,' ,0, 1). Prove the De-Morgan’s Law (x+y)' = x'.y'. (4)

13 a) Show that the lattice with three or fewer elements is a chain. (5)

b) What is Ring with Unity? Give an example of a commutative ring without unity. (4)

14 a) If the order of a group G be 'n' ie $a^n=e$ then the set $H = \{ a,a^{2,} ....,a^n\}$ forms a group with respect to the multiplication composition in G (5)

b) Define a bounded lattice. Give an example. (4)

PART E

Answer any four full questions, each carries10 marks.

15 a) Show the following implication without constructing the truth table (5)

$$((P \vee \urcorner P)\rightarrow Q)\rightarrow ((P \vee \urcorner P)\rightarrow R) \Rightarrow (Q\rightarrow R)$$

b) Negate the following statements and give the logical expression (5)

(i) All apples red. (ii) Some students are brilliant

16 a) Check the validity of the following argument: (5)

"If I get the job and work hard, then I will get promoted. If I get promoted , then I will be happy. Therefore ,either I will not get the job or I will not work hard."

b) Differentiate between free and bound variables with suitable examples. (5)

17 a) Derive the following using Rule CP if necessary (5)

(a) $\neg P \vee Q$, $\neg Q \vee R$, $R \rightarrow S \Rightarrow P \rightarrow S$ (b) $P \rightarrow (Q \rightarrow R)$, $Q \rightarrow (R \rightarrow S) \Rightarrow P \rightarrow (Q \rightarrow S)$

b) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$ using Indirect method of Proof . (5)

18 a) Show the following using indirect method of Proof (5)

$(R \rightarrow \neg Q)$, $R \vee S$, $S \rightarrow \neg Q$, $P \rightarrow Q \Rightarrow \neg P$

b) Show by Mathematical Induction that n3 + n is divisible by 3. (5)

19 a) An island has two tribes of natives. Any native from the first tribe always tells the truth, while the other tribe always lie.If you arrive at the island and ask a native if there is gold on the island. He answers " There is gold on the island if and only if I always tell the truth."Is there gold on the island? Justify your answer with the help of truth table. (5)

b) show that from (i)$(\exists x)(F(x) \wedge S(x)) \rightarrow (y)(M(y) \rightarrow W(y))$ and (5)

(ii)$(\exists y)(M(y) \wedge \neg W(y))$ the conclusion $(x)(F(x) \rightarrow \neg S(x))$ follows.

20 a) Show that $\neg(P \wedge Q) \rightarrow \neg P \vee (\neg P \vee Q)) \Leftrightarrow (\neg P \vee Q)$(without constructing truth table) (5)

b) Show that " If x is an integer and x^2 is even, then x is also even" by proof by contrapositive method. (5)

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