Reg No.:______________ Name:__________________________

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FIRST SEMESTER B.TECH DEGREE EXAMINATION(S), DECEMBER 2019

Course Code: MA101

Course Name: CALCULUS

Max. Marks: 100 Duration: 3 Hours

PART A

Answer all questions, each carries 5 marks. Marks

1 a) Find the sum of the series $\sum_{k=1}^{\infty} \frac{2}{3(k+1)}$ (2)

b) Determine whether the alternating series $\sum_{k=2}^{\infty} (-1)^k \frac{k}{k-1}$ converges. (3)

2 a) Find the slope of the function $f(x,y) = x\cos(xy) + y\sin(xy)$ at $(\pi,1)$ along the $\boldsymbol{x}$- direction. (2)

b) If $z = f(x^2-y^2)$, show that

$$y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = 0$$ (3)

3 a) Find $\lim_{t\to 0} \boldsymbol{r}(t)$, where $\boldsymbol{r}(t) = \langle 1+t^3, te^{-t}, \frac{\sin t}{t} \rangle$ (2)

b) Find the directional derivative of $f(x,y) = e^x \cos y$ at $P(0,\pi/4)$ in the direction of negative Y-axis (3)

4 a) Evaluate $\int_0^1\int_0^1\int_0^1 e^{x+y+z}\, dx\, dy\, dz$ (2)

b) Evaluate $\iint_R \left(x^2+y^2\right) dx\, dy$ where R is the region taken over the first quadrant for which $x+y\le 1$. (3)

5 a) Find the divergence of the vector field $F(x,y,z) = x^2y\, i + 2y^3z\, j + 3z\, k$ (2)

b) Evaluate $\int_c x^2dy + y^2dx$ where C is the path $y = x$ from (0,0) to(1,1) (3)

6 a) Determine the source and sink of the vector field $F(x,y,z) = 2(x^3-2x)i + 2(y^3-2y)j + 2(z^3-2z)k$ (2)

b) If S is any closed surface enclosing a volume V and if $A = axi + byj + czk$ prove that $\iint A.nds = (a+b+c)\,V$ (3)

PART B
Module 1
Answer any two questions, each carries 5 marks.

7	Test for convergence of the series $\sum_{k=0}^{\infty}\frac{(k!)^2}{(2k)!}$.	(5)
8	Find the radius of convergence of $\sum_{k=0}^{\infty}\frac{(2x-1)^k}{3^{2k}}$.	(5)
9	Expand $f(x) = sin\pi x$ into a Taylors series about $x = \frac{1}{2}$,up to third derivative.	(5)

Module 1I
Answer any two questions, each carries 5 marks.

10	If $u = f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$ find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.	(5)
11	Find the local linear approximation L(x ,y) of f(x ,y)=$\frac{1}{\sqrt{x^2+y^2}}$ at the point P(4,3).Compare the error in the approximation to f by L at the point Q(3.92,3.01) with the distance between P and Q.	(5)
12	Locate all relative extrema and saddle point for the function $f(x,y) = x^3 + y^3 - 6xy + 20.$	(5)

Module 1II
Answer any two questions, each carries 5 marks.

13	Find the equation of the unit tangent and unit normal to the curve $x = e^t \cos t, y = e^t \sin t, z = e^t$; $at\ t = 0.$	(5)
14	A particle moves along the curve $r(t) = \left(\frac{1}{t}\right)i + t^2 j + t^3 k$, where t denotes time. Find 1) The scalar tangential and normal components of acceleration at time$t = 1$. 2) The vector tangential and normal component of acceleration at time $t = 1$	(5)
15	Find the equation of the tangent plane and the parametric equations of the normal line to the surface $z = 4x^3y^2 + 2y - 2$ at (1,-2,10).	(5)

Module 1V
Answer any two questions, each carries 5 marks.

16	Use double integral to find the area of the plane enclosed by $y^2 = 4x\ and\ x^2 = 4y$	(5)
17	Change the order of integration to evaluate $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$	(5)

18 Use triple integral to find the volume of the solid with in the cylinder $x^2 + y^2 = 4$ and between the planes z = 0 and $y + z = 3$. (5)

Module V

Answer any three questions, each carries 5 marks.

19 If $\bar{r} = x\bar{\imath} + y\bar{\jmath} + z\bar{k}$ and $r = |\bar{r}|$, prove that $\nabla^2 r^n = n(n+1)\, r^{n-2}$ (5)

20 Evaluate $\int_C (3x^2 + y^2)\, dx + 2xy dy$ along the curve

$C: x = \cos t, y = \sin t,\ 0 \le t \le \frac{\pi}{2}$ (5)

21 Find the scalar potential of $\vec{F} = (2xy + z^3)\vec{\imath} + x^2\vec{\jmath} + 3xz^2\overline{k}$ (5)

22 Find the work done by $F(x, y) = (x + y)i + xy\, j - z^2\, k$ along the line segments from $(0,0,0)$ to $(1,3,1)$ to $(2,-1,5)$ (5)

23 Show that $\int_{(0,0)}^{(1,\frac{\pi}{2})} e^x \sin y\ dx + e^x \cos y\ dy$ is independent of path. Hence evaluate $\int_{(0,0)}^{(1,\frac{\pi}{2})} e^x \sin y\ dx + e^x \cos y\ dy$ (5)

Module VI

Answer any three questions, each carries 5 marks.

24 Evaluate using Green's theorem in the plane $\int_c (x^2 dx - xy dy)$ where C is the boundary of the square formed by $x = 0, y = 0, x = a, y = a$ (5)

25 Evaluate the surface integral $\iint_\sigma f(x, y, z)\ ds$ where $f(x, y, z) = x + y,\ \sigma$ is the portion of the surface $z = 6 - 2x - 4y$ in the first octant. (5)

26 Using divergence theorem find the flux across the surface σ which is the surface of the tetrahedron in the first octant bounded by $x + y + z = 1$ and the coordinate planes, $\bar{F} = (x^2 + y)\bar{\imath} + xy\bar{\jmath} - (2xz + y)\bar{k}$ (5)

27 Evaluate $\int_c (e^x dx + 2y dy - dz)$ where C is the curve $x^2 + y^2 = 4, z = 2$ using Stoke's theorem (5)

28 Evaluate the surface integral $\iint_\sigma f(x, y, z)\ ds$ where $f(x, y, z) = x^2 + y^2,\ \sigma$ is the surface of the sphere $x^2 + y^2 + z^2 = a^2$ (5)

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